convergence verification

synchrony to partial synchrony

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joint work with Mani Chandy and Concetta Pilotto

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consensus in distributed systems

system
n agents; agent i starts with value s_i

goal
agent i *converges* to value s_i^*

applications
consensus
pattern formation
flocking

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synchronous communication model

system executes in rounds $\cdots$

- in each round k, a subset (e.g., $\{1, 2, 4\}$), of agents are *active*
- each active agent read the current values of the other active agents to compute their new values
  - $s_2(k+1) = f_2(s_1(k), s_2(k), s_4(k))$
  - $s_2(k+1) = \frac{2}{3}s_1(k) + \frac{1}{3}s_4(k)$

strong assumption

- lock step synchrony
- instantaneous communication

partially synchronous (PS) communication

agent i sends a message with its value $s_i(t)$ at time t

- the message may be dropped
- otherwise, it is delivered to an *arbitrary subset* of agents within time $t + b$

b is a constant (system parameter) that is unknown to agents
PS model captures latency and uncertainties

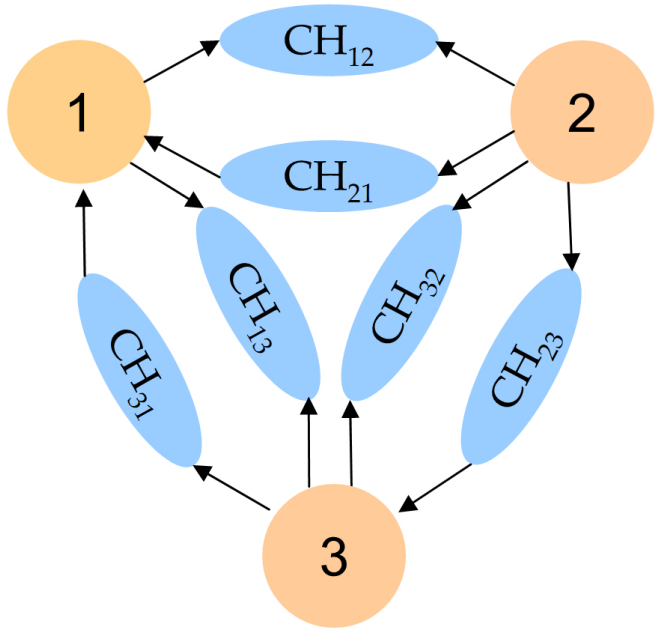
challenges in verifying PS systems

huge state space!

- $n(n-1)$ channels
- each channel stores (large number of) undelivered messages
- many possible message types as well

hence, limited success with model checking

generic techniques for compositional reasoning also not available



our main result[1]

Given the proof of convergence of a synchronous distributed system, under certain additional assumptions, it automatically guarantees the convergence of the corresponding PS system.

- proof of convergence: Lyapunov-like function
- additional assumptions about the structure of the lyapunov function
- correspondence defined by a natural transformation from synchronous to partially synchronous

. . . so what?

in designing algorithms for PS systems, if possible, verify the synchronous algorithm, check for additional conditions, and then use the natural transformation to get the PS system

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[1] To appear in Intl. Conf. on Formal Modelling and Analysis of Timed Systems (FORMATS'08)

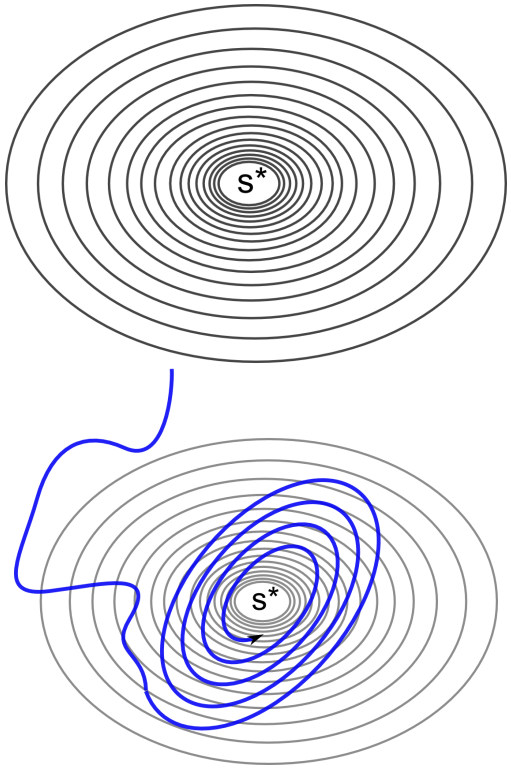
convergence

convergence to s^* is defined in terms of the set of neighborhood sets (topology around s^*) around s^*.

a discrete time system converges to a state s^* if for every neghborhood U around s^*, there exists $k_0 \in \mathbb{N}, k \geq 0$ such that for all $k > k_0$, $s(k) \in U$.

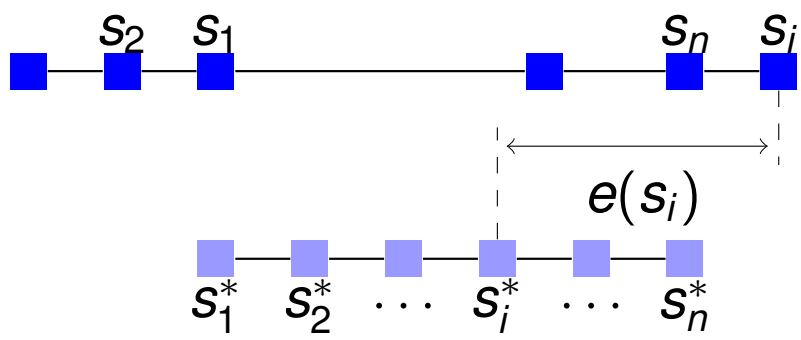
a continuous time system converges to a state s^* if for every neghborhood U around s^*, there exists $t_0 \in \mathbb{R}_{\geq 0}$ such that for all $t > t_0$, $s(t) \in U$.

convergence is a property of infinite executions



convergence in line formation

- s_i: position of i^{th} robot
- s_i^*: ideal position of i^{th} robot
- $e(s_i)$: deviation of i^{th} robot
- $U_p \triangleq$ states in which $\max_i e(s_i) \leq p$



fairness

some executions do not converge for silly reasons
e.g. an execution in which

- (*failure or perpetual message loss*) agent 3 is never active
- (*partition*) agents $\{1, 3\}$ never see $\{2, 4\}$

A *fairness condition* $\mathcal{F}$ restricts the set of executions by requiring that certain actions occur infinitely often.

e.g. “no permanent partition” is a fairness condition and we will look at convergence of executions that satisfy it.

proof of convergence

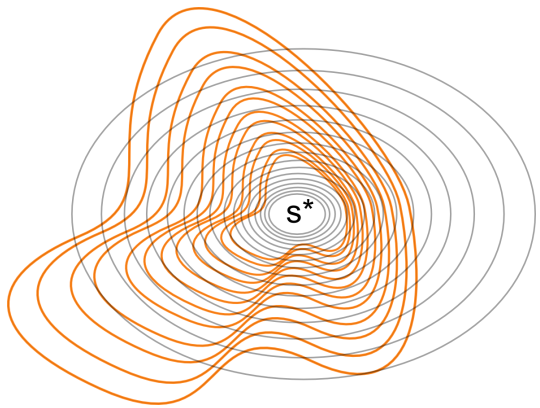
convergence is proved by establishing the existence of a (Lyapunov) function that is non-increasing along all executions of the system

Theorem (Tsitsiklis 1987)

Suppose there exists a collection of sets $\{P_k \subseteq S | \ k \in \mathbb{N}\}$ *satisfying:*

- *(monotonicity)* $k > l \Rightarrow P_k \subsetneq P_l$.
- *(granularity)* $\forall\ U, \exists\ k \in \mathbb{N}$, *such that* $P_k \subseteq U$.
- *(initial)* $S_0 \subseteq P_0$.
- *(invariance) if* $s \xrightarrow{a} s'$ *and* $s \in P_k$ *then* $s' \in P_k$.
- *(progress) If* $P_k \neq \{s^*\}$ *then there is a* $\mathcal{F}$*-fair action* a, $\forall\ s \in P_k, s' \in S$, $s \xrightarrow{a} s' \ \Rightarrow \ s' \in P_l$, *for some* $l > k$.

Then all $\mathcal{F}$*-fair executions of* $\mathcal{A}$ *converge to* s^*.



turns out to be necessary as well!

“additional assumptions”

simple assumption

For any $k \in \mathbb{N}$, P_k is of the form $p_{k1} \wedge p_{k2} \wedge \ldots p_{kn}$ where each p_{ki} is a predicate on the state of the i^{th} agent.

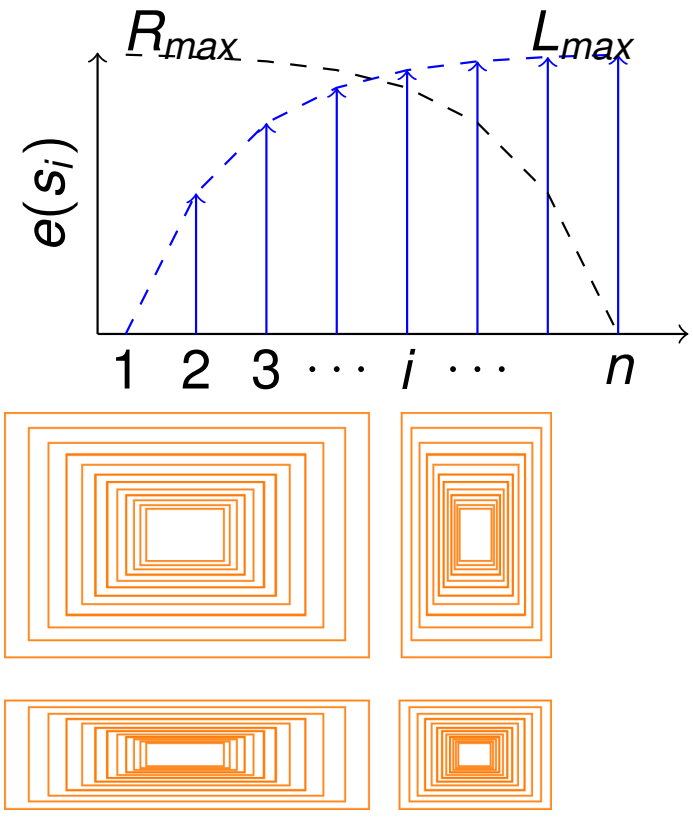
a more general assumption

appears in the paper

P_k's in line formation

$$L_{max}(s) \triangleq \forall l \quad e(s_l) \leq C\beta^m \left(1 - \frac{1}{2^{l-1}}\right)$$

$$R_{max}(s) \triangleq \forall r \quad e(s_r) \leq C\beta^m \left(1 - \frac{1}{2^{n-r}}\right)$$



transformation from synchronous to PS

variables

- $s_i : S := s_i(0)$
- $y_i : \texttt{Array}[[n] \to S_\perp]$ initially $\perp$

actions

- $\text{receive}_{ij}(m)$ sets $y_i[j] := m$
- $\text{update}_i(a)$ occurs periodically (a is chosen fairly) and it set $s_i := f_{ia}(x, y)$
- $\text{send}_i(m)$ occurs periodically

trajectories

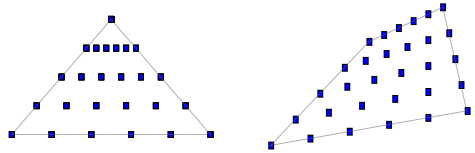
time progresses without violating task and message delivery deadlines

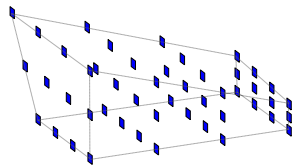
proof idea

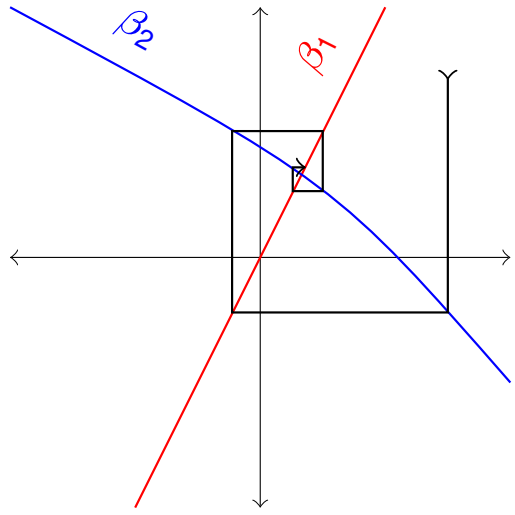
- from $\{P_k\}$ for $\mathcal{A}$ we construct $\{Q_k\}$ for $\mathcal{B}$ which satisfies all the conditions in Tistiklis' theorem
- define $\mathcal{R} \subseteq X \times S$, $\mathbf{x}\mathcal{R}s$ if and only if for each i one of the following holds
  - $\mathbf{x}.s_i = s_i$
  - there is j such that $\mathbf{x}.y_j[i] = s_i$
  - there is a message from i inbound to some j with value s_i
- monotonicity, granularity, and initial conditions are easy to check
- for invariance we use the additional assumption about $\{P_k\}$
- for progress we hae to use (a) additional assumptions about $\{P_k\}$, (b) the fairness of choosing the actions, (c) a weak assumption about message delivery.

applications

- **pattern formation.** in 2 and higher dimensions by simple extensions of the synchronous 1-D rule.
  agents with realistic dynamics
- **multi-player dynamic games.** each agent maximizes some utility based on its own state and (possibly old) messages from other agents
  do they agents converge to an equilibrium ?
- **aggregation and gossip protocols.** agents update their values using associative, commutative, and idempotent operations
  do they terminate ?
- **stationary distributions.** computing stationary distribution of big Markov chains using message passing over parallel processes (Miranker 1970s).







verification using theorem prover[2]

- there is an existing formalization of timed, untimed, and hybrid I/O automata in the PVS theorem prover
- these theories provide support for proving invariants and implementation relations—which requires only modelling and reasoning about finite executions
- to this existing body of theories we add notions of infinite executions, fairness, neighborhood sets, and prove the theorems for proving convergence
- several case studies

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[2] In proc. of Theorem Proving in High Order Logics (TPHOLs'08)

implementations[3]

- synthesis of Java programs for partially synchronous networks from synchronous (PVS) specifications
- currently using an arbiter for ensuring fairness

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[3] joint work with Brian Go and Jerome White submitted for review